Ramsification and Semantic Indeterminacy

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What if meaning is indeterminate?

Can we still (more or less) do classical semantics as we want to?

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- For many fragments of natural, mathematical, and scientific language, there is no uniquely determined intended interpretation.

  (E.g., mathematicians often understand $\mathfrak{I}(Prime)$ only to be determined uniquely on the natural numbers, but not beyond—the interpretation is *constrained* but there is *semantic indeterminacy*.)

In what follows, I will make a proposal for how to release the tension.

The key idea will be to apply a method known from scientific theory reconstruction to classical semantics: *Ramsification*.

Plan of the talk:

- A Sketch of Classical Semantics and Metasemantics
- The Challenge from Semantic Indeterminacy
- Ramsey Semantics to the Rescue
- Comparison with Supervaluationist and Classical Semantics
- Semantic Indeterminacy Reconsidered
- Conclusions

A Sketch of Classical Semantics and Metasemantics

(For simplicity, I will focus only on *extensional* semantics here.)

(1) $\mathcal{L}$: formalized fragment of natural, mathematical, or scientific language.

(2) Interpretation: An interpretation F of $\mathcal{L}$ assigns references/extensions to the members of the descriptive vocabulary of $\mathcal{L}$ over a domain $Uni(F)$.

(I leave open whether the metavariable 'F' is first-order or higher-order.)

(3) Satisfaction: Define '$F \models A$' in a recursive Tarskian manner.

E.g.: $F \models P(a)$ if and only if $F(a) \in F(P)$.
$F \models \neg A$ if and only if it is not the case that $F \models A$.
$F \models A \vee B$ if and only if $F \models A$ or $F \models B$.
$\vdots$

(4) Logical consequence:

$A_1, \ldots, A_n \models C$ if and only if for all F: if $F \models A_1, \ldots, A_n$, then $F \models C$.

(5) Intended/admissible interpretations: Amongst all interpretations F of $\mathcal{L}$, there is a subclass *Adm* of *intended* or *admissible* interpretations of $\mathcal{L}$ that is determined jointly by

(i) all linguistic facts concerning the competent usage of predicates and singular terms in $\mathcal{L}$ (e.g.: the definition of *Prime*),

(ii) all non-linguistic facts that are relevant as to whether the atomic formulas in $\mathcal{L}$ are satisfied (e.g.: what satisfies the definiens for *Prime*),

(iii) where determination is governed by metasemantic laws that concern the atomic formulas, and hence the predicates and singular terms, of $\mathcal{L}$.

Adm may be thought of as a theory in the non-statement view of theories: a class of interpretations F satisfying (metasemantic) constraints on F.

Here is one example of (iii) above (cf. Putnam 1975):

for all kind terms K in $\mathcal{L}$, for all objects d and d': if the meaning of K was collectively specified in $\mathcal{L}$ by pointing at d while being interested in the physical structure of d, and d' has the same physical structure as d, then: if $d' \in \mathit{Uni}(F)$ then $d' \in F(K)$.

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(6) The central presupposition of classical (meta-)semantics:

Adm has exactly one member: $Adm = \{\mathfrak{I}\}$.

The uniquely determined intended interpretation $\mathfrak{I}$ is meant to convey the intended interpretations of predicates and singular terms, the intended universe of discourse $Uni(\mathfrak{I})$, and the actual truth values:

(7) Classical truth: For all sentences *A* in $\mathcal{L}$: *A* is true if and only if $\mathfrak{I} \models A$.

(1)–(7) constitute the classical semantic/metasemantic package.

Its main problem is: it is dangerous to presuppose (6), since it might be *false*.

- Semantic indeterminacy:

  What if meaning is indeterminate, that is, *Adm* has *more* than one member?

The Challenge from Semantic Indeterminacy

- Example 1: Vagueness (cf. Lewis 1986)

  Let $\mathcal{L}$ formalize a fragment of natural language with the predicate *Bald*.

  We only know partially what *Adm* is like: if F is in *Adm*, then $0 \in F(Bald)$, $100000 \notin F(Bald), \ldots$

  But it seems unlikely that $Adm = \{\mathfrak{I}\}$, such that for every number n,

  – either the metasemantic constraints determine that $n \in \mathfrak{I}(Bald)$,
  – or they determine that $n \notin \mathfrak{I}(Bald)$

  (where n is the number of hairs on the head of a corresponding person).

  For, at least prima facie, it is plausible that there are borderline cases n to which one may competently ascribe *Bald*, but to which one may also competently refrain from ascribing *Bald* and indeed ascribe $\neg Bald$.

  Thus, there is more than one intended/admissible interpretation in *Adm*.

- Example 2: Arithmetic (cf. Benacerraf 1965)

  Let $\mathcal{L}$ be the language of second-order arithmetic with N, 0, s, $+$, $\cdot$.

  If one is a (set-theoretic) structuralist about arithmetic, then the interpretation of arithmetical symbols is only determined up to isomorphism:

$$Adm = \{F : F \models PA_2\},$$

  where PA_2 is the second-order Dedekind-Peano axioms for arithmetic (that is: $F(0) \in F(N)$; for all d, if $d \in F(N)$ then $F(s)(d) \in F(N)$;...).

  But there are infinitely many pairwise isomorphic set-theoretic interpretations F of $\mathcal{L}$ that satisfy PA_2.

  Hence, *Adm* includes more than one intended/admissible interpretation.

  (So this is a case of semantic indeterminacy without vagueness. Note that even *non-set-theoretic* structuralists would have to accept the existence of semantic indeterminacy in mathematics.)

- Example 3: Conceptual Progress (cf. Field 1973)

  Let $\mathcal{L}$ be the language of Newtonian mechanics with the term *mass*.

  Using the language of modern relativistic mechanics, there seem to be two interpretations of *mass*, such that there is no fact of the matter which of them delivers "the right" intended reference of Newtonian *mass*:

  – $\mathfrak{I}_1(\textit{mass})$ coincides with relativistic mass (total energy/c^2),

  – $\mathfrak{I}_2(\textit{mass})$ coincides with proper mass (non-kinetic energy/c^2),

  which can come apart in value but which are both maximally charitable (saving some of Newton's central claims but not all of them).

  So *Adm* includes more than one intended/admissible interpretation.

  (This is another case of semantic indeterminacy without vagueness.)

One possible reaction: Epistemicism (e.g. Williamson 1992)

- We should stick to classical semantics and metasemantics: $Adm = \{\mathfrak{I}\}$.
- It is just that we do not *know* how the metasemantic facts determine $\mathfrak{I}$:

  No, vague terms are not semantically indeterminate: their vagueness can be explicated otherwise (e.g. counterfactually), and complete extensions are somehow determined by the facts even though we do not know how.

  No, structuralism about arithmetic is wrong: there is more to arithmetical terms than their structural content, whatever it is exactly.

  No, 'mass' as used by Newtonian physicists does have unique reference, even when it is hard to say what it refers to.

  ⋮

  (No, Quine's, Putnam's, Kripke's, Feferman's, Wilson's,... arguments for semantic indeterminacy do not succeed either.)

Can epistemicists deliver such a piecemeal defence? Questionable!
Ramsey semantics will also undermine an abductive defence of classicism.

A second possible reaction: Supervaluationism
(cf. van Fraassen 1966, Fine 1975, McGee & McLaughlin 1994, Keefe 2000)

- Do not assume that $\mathit{Adm} = \{\mathfrak{I}\}$.

  (So allow for semantic indeterminacy!)

- But change classical truth to “super-truth”:

  For all sentences A in $\mathcal{L}$: A is true if and only if for all F in Adm, $F \models A$.

But then semantics is not compositional anymore, and there are logical problems: e.g., if one extends the language by a ‘determinately’ operator *Det* for super-truth and when logical consequence is super-truth-preservation,

$$A \models_{SV} \mathit{Det}(A), \text{ but not for all } A\text{: } \models_{SV} A \rightarrow \mathit{Det}(A).$$

$\hookrightarrow$ Some of the metarules of classical logic would fail! (cf. Williamson 1994)

Ramsey Semantics to the Rescue

Here is an alternative proposal that will preserve classical truth and logic:

- Accept the classical package except for assuming that $Adm = \{\mathfrak{I}\}$:

  Allow for semantic indeterminacy!

- Take the terms '$\mathfrak{I}$' ("intended interpretation") and 'true' from classical semantics to be theoretical terms that are introduced by

$$\mathfrak{I} \in Adm \text{ and for all sentences } A \text{ in } \mathcal{L}: A \text{ is true iff } \mathfrak{I} \models A.$$

- Replace '$\mathfrak{I}$' and 'true' by variables and add existential quantifiers:

$$\exists F, T\colon\ F \in Adm \text{ and for all sentences } A \text{ in } \mathcal{L}: A \in T \text{ iff } F \models A.$$

  In other words: *Ramsify classical semantics!* (cf. Ramsey 1929)

  (Note that '$\exists F(F \in Adm \ldots$' has wide scope over truth!)

Alternatively: Define "the" intended interpretation by means of a primitive metalinguistic *epsilon operator* and thereby preserve the terms '$\mathfrak{I}$' and 'true'.

- Hilbert (1922) (and Ackermann 1924, Hilbert and Bernays 1934): ε-terms as a tool in metamathematics, denoting "ideal elements".

- If A is a formula and x is a variable, then $\varepsilon x A$ is a term. (*In*definite description: "*an* x, such that A".)

- Add logical axioms for the ε-operator:

  $\exists x A[x] \leftrightarrow A[\varepsilon x A[x]]$
  Extensionality: $\forall x(A[x] \leftrightarrow B[x]) \rightarrow \varepsilon x A[x] = \varepsilon x B[x]$

- Do not assume that the metasemantic facts determine what ε "picks"! (Contrast: Breckenridge and Magidor 2012 on "arbitrary reference".)

- Amend classical semantics by postulating:

  (i) $\exists F(F \in Adm)$, (ii) $\mathfrak{I} = \varepsilon F(F \in Adm)$ (entailing that $\mathfrak{I} \in Adm$),

  (iii) for all sentences A in $\mathcal{L}$: A is true if and only if $\mathfrak{I} \models A$.

Comparison with Supervaluationist and Classical Semantics

Unlike supervaluationism, Ramsey semantics still predicts:

- Truth is classical (i.e., classical truth biconditionals and compositionality):

  $P(a)$ is true if and only if $\mathfrak{I} \models P(a)$
  if and only if $\varepsilon F(F \in Adm) \models P(a)$
  if and only if $\varepsilon F(F \in Adm)(a) \in \varepsilon F(F \in Adm)(P)$.

  $\neg A$ is true if and only if $\mathfrak{I} \models \neg A$
  if and only if $\mathfrak{I} \not\models A$
  if and only if it is not the case that A is true.

  $A \vee B$ is true if and only if $\mathfrak{I} \models A \vee B$
  if and only if $\mathfrak{I} \models A$ or $\mathfrak{I} \models B$
  if and only if A is true or B is true. Etc.

- Logic is fully classical:

  Logical consequence is still preservation of truth, and all classical rules/metarules remain logically valid even with a 'determinately' operator.

Indeed, Ramsey semantics is *almost* like classical semantics:

Ramsey semantics leads deductively to the same predictions as classical semantics so long as the classical semanticist does not explicitly invoke their (usually tacit!) assumption of semantic determinacy, that is, $\mathit{Adm} = \{\mathfrak{I}\}$.

At the same time, Ramsey semantics is *less risky* than classical semantics:

- unlike classical semantics, it does *not* presuppose a uniquely metasemantically determined intended interpretation;
- instead, it merely presupposes that there exists a classical interpretation F that conforms to all existing metasemantic constraints and from which truth is defined by means of the classical semantic rules;
- in the case of semantic determinacy, Ramsey semantics coincides with classical semantics; but it *allows for* semantic indeterminacy.

Semantic Indeterminacy Reconsidered

Example 1: Vagueness reconsidered

- If one can derive in the metatheory that for all F in Adm it holds that

  $$\text{for all } m, n, \text{ if } m > n \text{ and } m \in F(Bald) \text{ then } n \in F(Bald),$$

  then Ramsey semantics will derive from this (just as supervaluationism)

  $$\text{for all } m, n, \text{ if } m > n \text{ and } Bald(m) \text{ is true, then } Bald(n) \text{ is true}$$

  and therefore also, e.g., (unlike supervaluationism!)

  $$\text{for all } m, n, \text{ if } m > n \text{ and } \neg Bald(m) \text{ is not true, then } Bald(n) \text{ is true.}$$

- The Sorites argument

  $Bald(0)$
  For all n: if $Bald(n)$ then $Bald(n+1)$
  Therefore, $Bald(100000)$.

  is logically valid but not sound: premise 2 is not true in Ramsey semantics.

- Hence,

  there exists an *n*, such that $Bald(n)$ and $\neg Bald(n+1)$

  is true.

- In fact, every *F* in *Adm* satisfies that existential claim, and thus

  Det(there exists an *n*, such that $Bald(n)$ and $\neg Bald(n+1)$).

  is true.

- But this does *not* entail that

  there exists an *n*, such that $Det(Bald(n)$ and $\neg Bald(n+1))$

  is true (cf. McLaughlin and McGee 1994).

  In that sense, Ramsey semantics is *not* committed to sharp boundaries!

- *Bald* has a unique intended interpretation ($\exists! X(X = \Im(Bald))$)
  but not a *factually* uniquely determined intended interpretation.

In a nutshell:

- Classical semantics assumes the borderline facts to be *complete* (one can derive the truth of $\exists n Det(Bald(n) \& \neg Bald(n+1))$) and employs a *classical* concept of truth.

- Supervaluationist semantics does *not* assume the borderline facts to be *complete* (one cannot derive the truth of $\exists n Det(Bald(n) \& \neg Bald(n+1))$) but invokes a *non-classical* concept of truth.

- Ramsey semantics does *not* assume the borderline facts to be *complete* (one cannot derive the truth of $\exists n Det(Bald(n) \& \neg Bald(n+1))$) but uses a *classical* concept of truth.

According to Ramsey semantics, 'is metasemantically determined' may differ in extension from 'true': semantics may outrun the facts!

We talk *as if* the metasemantic facts had determined a unique interpretation.

(For higher-order vagueness: apply Ramsification/metalinguistic epsilon terms *at every level throughout the Tarskian hierarchy!*)

Example 2: Arithmetic reconsidered

- Let again $Adm = \{F : F \models PA_2\}$,

  where PA_2 is the second-order Dedekind-Peano axioms for arithmetic.

- Thus, $\exists F(F \in Adm)$.

- Ramsey semantics adds to this:

  $\Im = \varepsilon F(F \in Adm)$.

  For all sentences A in $\mathcal{L}$: A is true if and only if $\Im \models A$.

  (cf. Shapiro 2012 on structural terms as "parameters".)

- In Ramsey semantics the following holds by ε-logic and definitions:

$$PA_2 \text{ is true iff } \exists F(F \in Adm),$$

  that is,

$$PA_2 \text{ is true iff } PA_2 \text{ is satisfiable.}$$

  (cf. Hilbert vs Frege)

Example 3: Conceptual Progress reconsidered

- Let again $Adm = \{\Im_1, \Im_2\}$, where
  - $\Im_1(mass)$ coincides with relativistic mass (total energy/c^2),
  - $\Im_2(mass)$ coincides with proper mass (non-kinetic energy/c^2).
- Thus, $\exists F(F \in Adm)$.
- Ramsey semantics adds to this:

  $\Im = \varepsilon F(F \in Adm)$.

  For all sentences A in $\mathcal{L}$: A is true if and only if $\Im \models A$.
- Hence, Ramsey semantics for Newtonian mechanics derives that

$$\Im(mass) = \Im_1(mass) \text{ or } \Im(mass) = \Im_2(mass),$$

without deriving either disjunct.

- Ramsey semantics is consistent with referential (diachronic or synchronic) continuity between two scientific languages $\mathcal{L}_1$ and $\mathcal{L}_2$, that is, with

  $$\varepsilon F(F \in Adm_1) = \varepsilon F(F \in Adm_2),$$

  so long as $Adm_1 \cap Adm_2 \neq \emptyset$.

  It is consistent to understand terms from the two languages *as if* the metasemantic facts had determined their reference to be the same.

cf. Carnap (1959, 1961) on defining scientific terms with help of epsilon terms:

> *So this definition [gives] just so much specification as we can give, and not more. We do not want to give more, because the meaning should be left unspecified in some respect, because otherwise the physicist could not—as he wants to—add tomorrow more and more postulates... and thereby make the meaning of the same term more specific than [it is] today. So, it seems to me that the* ε*-operator is just exactly the tailor-made tool that we needed, in order to give an explicit definition, that, in spite of being explicit, does not determine the meaning completely... (Carnap 1959)*

Conclusions

- If one wants to be prepared for semantic indeterminacy—unlike epistemicism—and if one also aims to stay much closer to classical semantics than supervaluationism, then one ought to *Ramsify classical semantics*.
- In mathematics, intuitionists have always been sensitive to these kinds of worries: in order for $A \vee \neg A$ to be true, it seems that some facts would have to determine that A or some facts would have to determine that $\neg A$.

  Not so, if mathematical language is interpreted by Ramsey semantics!
- Similarly, understanding scientific language and language-change in a "realist" interpretational semantics might seem to require strong scientific/metaphysical assumptions (that is, semantic determinacy).

  Not so, if scientific language is interpreted by Ramsey semantics!

Initially, the following statements seemed true (subject to qualifications) and yet they also seemed in tension with each other:

- Classical semantics successfully reconstructs the (truth-conditional) meaning of natural, mathematical, and scientific language.

- Classical semantics presupposes the existence of a uniquely determined intended interpretation.

- For many fragments of natural, mathematical, and scientific language, there is no uniquely determined intended interpretation.

$\hookrightarrow$ The tension is released by understanding that the Ramsification of classical semantics is sufficient for a successful reconstruction of meaning.

$\hookrightarrow$ And the same Ramsification no longer presupposes the existence of a uniquely determined intended interpretation. ✓

What if meaning is indeterminate?

Don’t worry: even if so, we may still *almost* do classical semantics.