

Methodological Frames: Mathematical structuralism and proof theory

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Abstract. Mathematical structuralism is deeply connected with Hilbert and Bernays' proof theory and their programmatic aim to ensure the consistency of mathematics. That goal was to be reached on the sole basis of finitist mathematics. Gödel's second incompleteness theorem forced a step from *absolute finitist* to *relative constructivist* proof theoretic reductions. The mathematical step was accompanied by philosophical arguments for the special nature of the grounding constructivist frames. Against this background, I examine Bernays' reflections on proof theoretic reductions – from the mid-1930s to the late 1950s and beyond – that are focused on narrowly arithmetic features of frames.

I propose a more general characterization of frames that has ontological and epistemological significance. It is rooted in the internal structure of mathematical objects and is given in terms of accessibility: *domains of objects* are accessible if their elements are inductively generated; *principles for such domains* are accessible if they are grounded in our understanding of the generating processes. The accessible principles of inductive proof and recursive definition determine the generated domains uniquely up to a canonical isomorphism. The determinism of the inductive generation allows us to refer to the objects of an accessible domain; at the same time, the canonicity of the isomorphism justifies an "indifference to identification".